Math 300 / Honors 213

First Hour Exam

Name ________________________________

Friday, Feb. 10
90 points (will be normalized to 100 pts. in the gradebook)
I. Some definitions (5 points each). Give formal definitions of the following. In the case of the "new" definition (c), make sure that you use only terms defined in chapter I of the textbook:

a. A ray

b. A midpoint of a segment

c. A parallelogram (recall the standard high school definition of a four-sided figure with opposite sides parallel).

d. A circle.
II. 1. (5 pts) In a proof of

\[ S_1 \]
\[ S_2 \]
\[ S_3 \]
\[ \ldots \]
\[ S_N \]
\[ \ldots \]
\[ \Box C \]

What must be true of \( S_1 \) .. \( S_N \) and \( C \) for \( C \) to follow logically from those statements?

2. (10 pts.) What is a RAA proof? How do you set about proving something using an RAA proof?
III. (5 pts. each). Given the statement "If I drink too much coffee I have trouble sleeping". Give, in ordinary English:

a. The converse to the statement

b. The contrapositive of the statement

c. The negation of the statement

d. What is the sufficient part?

e. What is the necessary part?
V.   (5 pts. each) Writing the negation of a statement is a useful thing to be able to do. In part (a), write down the negation of the statement, simplifying it so that negations occur only before predicates. In part (b), state the negation in conversational English.

a. \( \square x (p(x) \square q(x)) \)

b. Every line has at least two distinct points.

(We'll have more practice on these sorts of questions in the exercise set to chapter 2)

VI  (15 pts.) One of the rules of logic asserts that the negation of an if-then statement if \( p \) then \( q \) has the form \( p \) and (not \( q \)),

\[ \square(p \square q) = (p \square \square q) \]

Use truth-tables to demonstrate this (the following diagram should get you started). What in the completed truth table tells you that you should believe the statement?

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p =&gt; q</th>
<th>not (p =&gt; q)</th>
<th>not q</th>
<th>p and (not q)</th>
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VII (5 pts.) Say something (a brief sentence - something relevant to the course) about three of the following names.

a. Euclid
b. Thales of Miletus
c. David Hilbert
d. Proclus Diadochus
e. Gottlob Frege
f. Adrien Marie Legendre
g. Kurt Gödel
h. Pythagoras of Samos