General Notes:

1. **Show work.**
2. Look over the test first, and then begin.
3. Calculators are permitted on this exam, but only for basic arithmetic (i.e., no built-in statistical calculations)

Friday, December 2, 2011
100 pts.
I. Some short questions (5 pts. each):
   
   a. State the Central Limit Theorem for the sampling distribution of a sample average \( \bar{x} \).
   
   b. What is a level C confidence interval for a population mean \( \mu \)?
   
   c. Exactly what does it mean to say that results are significant at the \( \alpha=0.05 \) level?
   
   d. What are the null and alternate hypotheses, and what roles do they play in hypothesis testing?
   
   e. Who is William Gossett and what does he have to do with statistics?
I. Binomial Distributions (5 pts. each)

1. What is a binomial setting? That is, what must be true in order to use a binomial distribution?

2. Suppose we have a binomial distribution B(10, 0.4). What do the numbers 10 and 0.4 indicate?

3. For the binomial distribution above, what are the mean $\mu$ and the standard deviation $\sigma$ of the binomial count of successes for this distribution? Your answers should be numbers.

4. Use Table C to find the probability of three or fewer successes in the binomial distribution above (i.e., find $P(X \leq 3)$).
II. Confidence intervals (5 pts. each)

A SRS of size 25 is taken from a population and are given a test with scores ranging from 0 to 100, and the average score is calculated to be 75. Suppose that we know that the standard deviation of test scores is $\sigma = 12.76$. We want to calculate a 95% confidence interval for the mean test score in the population.

1. Calculate the margin of error in this case. Please remember that in this and in all other questions on this exam to show your work. An answer (even a correct one) without work shown may not receive credit.

2. Give the 95% confidence interval as an interval.

3. Suppose that we want to test a null hypothesis $H_0 : \mu = 72$ against the alternate hypothesis $H_a : \mu \neq 72$ at the $\alpha=0.05$ level of significance. Given the work above, does this sample provide enough evidence to reject the null hypothesis? Why or why not.
III. Tests of significance (5 pts. each)

A. Here is some data from a randomly selected exam (not from this class or year):

<table>
<thead>
<tr>
<th>N</th>
<th>Mean</th>
<th>SE Mean</th>
<th>StDev</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>77.09</td>
<td>2.49</td>
<td>11.68</td>
</tr>
</tbody>
</table>

1. How many degrees of freedom do we have?

2. Using Student’s t distribution, calculate a 95% confidence interval, giving your answer with a margin of error. Please remember to show details of your calculation.

3. We want to test $H_0: \mu = 70$ against $H_a: \mu \neq 70$. Give the formula for the appropriate t test statistic and calculate it.

(continued on the following page)
4. Use table D to find the P-value for this test of significance. Table D will not give you a precise answer. Pick the larger of the two P-values that bracket your calculated t-value.

5. Do we reject $H_0$ at the $\alpha=0.05$ level? Why or why not?

B. (5 pts.) Suppose that we are comparing samples from two independent populations and find

<table>
<thead>
<tr>
<th>Population</th>
<th>N</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$n_1$</td>
<td>$\bar{x}_1$</td>
<td>$s_1$</td>
</tr>
<tr>
<td>2</td>
<td>$n_2$</td>
<td>$\bar{x}_2$</td>
<td>$s_2$</td>
</tr>
</tbody>
</table>

1. What is the form of the t-test statistic to compare the null hypothesis $H_0 : \mu_1 = \mu_2$ against the alternate hypothesis $H_a : \mu_1 \neq \mu_2$? (That is, write the formula for $t$ in this case)
C. (15 pts. – an essay question)

Suppose that we are doing a test of significance testing a null hypothesis $H_0$ against an alternate hypothesis $H_a$. Suppose further that we have selected $\alpha=0.01$ as our level of significance and calculate a $P$-value of 0.003 from a simple random sample. As if writing a letter to an intelligent friend who has not (yet) taken statistics, explain carefully why we feel that this result provides evidence for rejecting the null hypothesis in favor of the alternate hypothesis. Please note that simply comparing the values of $\alpha$ and $P$ is not sufficient. Explain to your friend why we should believe this result.