I. Theory

1. (10 pts.) Give a formal definition of \( \int_a^b f(x) \, dx \), and then give an informal definition as you would use in explaining it to a fellow student.
2. (10 pts.) We have studied two theorems called “the fundamental theorem of calculus”. Give a formal statement of one, and explain why it might be considered surprising.

3. (5 pts.) Give a formal definition of \( \lim_{n \to \infty} a_n = L \).

4. (5 pts.) Give a formal definition of \( \prod_{n=1}^{\infty} a_n = L \).
6. (10 pts.) What do we mean (formally) by saying that a series is a Taylor series expansion of the function $f(x)$ about $x=c$?
II. **Techniques (10 points each):** Evaluate the following definite and indefinite integrals. Show how you got your answer.

1. \[ \int_{0}^{1} (2x^2 + x + 1)^2 (4x + 1) \, dx \]

2. \[ \int \tan^2(x) \sec^2(x) \, dx \]

3. \[ \int_{0}^{\frac{\pi}{4}} \cos^2(x) \, dx \text{ (limits are 0 to } \frac{\pi}{4}) \]
4. \[ \frac{dx}{(x + 1)(x - 2)} \]

5. \[ \cos(x)e^x \, dx \]

6. \[ \frac{dx}{x^2} \]
8. (15 pts.) Use the trapezoid rule with $n = 4$ to approximate $\int_{0}^{1} x^2 \, dx$. Carry your answer to the point where only numbers remain. (i.e.: To the point at which you could really use a calculator).
III. Applications

1. (15 pts.) Calculate the volume of the solid formed by rotating the region between the curves $y = x$ and $y = x^2$ for $0 \leq x \leq 1$ about the x-axis.
2. (10 pts) Calculate the arc length of the curve \( y = x^{\frac{1}{2}} \) for \( 0 \leq x \leq 4 \). Set up only (i.e., take the calculation to the point at which only an integration needs to be done). 5 pts. extra credit: Finish the calculation.
3. (10 pts.) State the theorem of Pappus, and use it to find the volume of the region formed by rotating the circle of radius 1 centered about the point \((2,0)\) about the y-axis.
4. (15 pts.) Solve the differential equation \( \frac{dQ}{dt} = 3Q(t) \) subject to the condition that \( Q(t) = 5 \) when \( t = 0 \). How long does it take to reach \( Q(t) = 10 \)?
IV  Power series

1.  (5 pts each). Say whether the following two infinite series converges or diverges, and say why

   a. \[ \sum_{k=1}^{\infty} \frac{(k + 1)^2}{k^2} \]

   b. \[ \sum_{k=2}^{\infty} \frac{1}{k \ln k} \]
2. (10 pts.) Write down the first three terms of the Maclaurin series for 
f(x) = e^{x^2}.

3. (10 pts.) What is the convergence set and the radius of convergence of the 
series \( \sum_{k=1}^{\infty} \frac{(3x)^k}{2^{k+1}} \)?