Math 122C

FINAL EXAM

NAME__________________________________________________________

Monday, Dec. 12

200 pts.
I. Theory

1. (10 pts.) Give a formal definition of \( \int_a^b f(x)\,dx \), and then give an informal definition as you would use in explaining it to a fellow student.
2. (10 pts.) We have studied two theorems called “the fundamental theorem of calculus”. Give a formal statement of one, and explain why it might be considered surprising.

3. (5 pts.) Give a formal definition of $\lim_{n \to \infty} a_n = L$.

4. (5 pts.) Give a formal definition of $\prod_{n=1}^{\infty} a_n = L$.
6. (10 pts.) What do we mean (formally) by saying that a series is a Taylor series expansion of the function $f(x)$ about $x=c$?
II. Techniques (10 points each): Evaluate the following definite and indefinite integrals. Show how you got your answer.

1. \( \int_{0}^{1} (2x^2 + 1)(x^3 + x)^2 \, dx \)

2. \( \int \sin^2(x) \cos(x) \, dx \)

3. \( \int \sin^2(x) \cos^2(x) \, dx \)
4. \[ \int_{0}^{1} \frac{dx}{1 + x^2} \]

5. \[ \int e^x \, dx \]

6. \[ \int \frac{dx}{x^2} \]
8. (15 pts.) Use Simpson's rule with $n = 4$ to approximate $\int_0^1 x^2 \, dx$. Carry your answer to the point where only numbers remain. (i.e.: To the point at which you could really use a calculator).
III. Applications

1. (15 pts.) The cross section of a trough is a right triangle with top 5', depth 10', and length 10'. If it is filled with water (\( \rho = 62.4 \text{ lb/ft}^3 \)) how much work is involved in pumping the water out?
2. (10 pts) Calculate the x-coordinate of the region bounded above by \( y = x \) and below by \( y = x^2 \) for \( 0 \leq x \leq 1 \).
3. (10 pts.) The force exerted by a spring is $F = -3x$ where $x$ is the extension from some rest point $x=0$ inches. How much work is done by stretching the spring from its rest position ($x = 0$) to $x = 1$?
4. (15 pts.) Solve the differential equation \( \frac{dQ}{dt} = 3(10 \Box Q) \) subject to the condition that \( Q = 9 \) when \( t = 0 \).
IV Power series

1. (5 pts each). Say whether the following two infinite series converges or diverges, and say why

a. \[
\sum_{k=2}^{\infty} \frac{1}{k \ln k}
\]

b. \[
\sum_{k=1}^{\infty} \frac{k + 1}{(k + 2)^3}
\]
2. (10 pts.) Write down the first three terms of the Maclaurin series for 
\( f(x) = e^{\sin(x)} \).

3. (10 pts.) What is the [convergence set and] radius of convergence of the 
series \( \sum_{k=1}^{\infty} \frac{k^2 x^k}{k+1} \)?