General Notes:

1. **Show work.**
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam. Carry out any calculations to the point at which you would need a calculator (for example, to take the square root of the logarithm of a number) and leave it in that form.

   Tuesday, November 16, 2010
   90 pts. (will be adjusted to 100 points in the gradebook)
I. Inverses (5 pts each)

a. Find \( \text{ArcCos}(\frac{\sqrt{3}}{2}) \) (inverse cos)

b. \( \frac{d}{dx} \text{ArcTan}(x) = \)

2. Chain Rule (5 pts. each)

\[ \frac{d}{dx} (x^2 + 2x + 3)^{15} = \]

\[ \frac{d}{dx} e^{\sin(x)} = \]

\[ \frac{d}{dx} e^{\ln(x)} = \]
2. Approximations and rates of change

a. (5 pts.) What is the standard linear approximation to the function $f(x) = \sqrt{1+x}$ at the point $x = 0$? Use it to approximate the square root of 1.01. (problems 16, 17, page 218)

b. (10 pts.) An object is dropped from the top of a 100 meter high tower. Its height above the ground after $t$ seconds is $100 - 4.9t^2$. How fast is it following 9 seconds after it is dropped? What is its acceleration at the time? What is its jerk at that time?
(15 pts.) Sand is being dropped by a conveyor belt onto a conical pile which is always twice as high as the base (a circle) is wide at the rate of 10 cubic feet / minute. How fast is the height of the pile increasing when the height is 18 feet (and the base has a diameter of 9 feet)? Recall that the volume of a right circular cone is given by $V = \frac{\pi}{3} r^2 h$. Hint: use similar triangles.
3. Mean Value Theorem and some applications.

a. (5 pts.) State the Extreme Value Theorem (with preconditions)

b. (5 pts.) State the Mean Value Theorem (with preconditions)

c. (5 pts.) Verify the mean value theorem for the function $y = x^2 + 1$ on the interval $[1, 2]$ (i.e., find a number $c$ in the interval $(1, 2)$ which satisfies the conclusion of the mean value theorem in this case.)
Implicit differentiation

a. (20 pts.) Find the equation of the lines tangent and normal to the curve $x^2 + y^2 = 4$ at the point $(1, \sqrt{3})$ by first finding the slope at that point $\frac{dy}{dx}$ using implicit differentiation and then using the slope and the point to find the tangent and normal lines at that point.