I. Optimization

1. A field is to be fenced using 100' of fencing. Happily, a river with an unusually straight bank takes up one side of the field. Using the 100' feet of fencing to fence the remaining three sides of a rectangular field, we want to find the dimensions of the largest area that can be enclosed.

   a. (5 pts.) Sketch a picture of the situation, labeling what you can.

   b. (15 pts.) Find the dimensions of the largest rectangular field we can enclose under these conditions.
II. l'Hôpital's rule

1. (5 pts. each) Use l'Hôpital's rule to find the following limits. Show your work:

\[ \lim_{x \to 0} \frac{2x^2 - 3x + 1}{4x^2 - 17x + 15} \]

\[ \lim_{x \to 0^+} x^4 \quad \text{(problem 55)} \]

III. Newton's Method

1. (10 pts.) Suppose that we want to solve the equation \( x^3 - x = 0 \). We use Newton's method with an initial guess of \( \frac{1}{2} \). What is the next guess?
IV. Antiderivatives (5 pts each) Find the following antiderivatives. Remember the constant of integration!

\[ \int (x^3 + 3x^2 - 7x + 1) \, dx \]

\[ \int \sinh(x) \, dx \]

\[ \int e^x \, dx \]

\[ \int \cos(x) \, dx \]

\[ \int \frac{dx}{1 + x^2} \]
V. Summations

1. (10 pts.) Evaluate $\sum_{k=1}^{10} (2k - 1)$ to a number using the rules and formulae we have developed. Show your work.
VI. The definite integral

1. (15 pts.) Give a careful definition of a Riemann Sum for a function $f$ on an interval $[a,b]$, explaining all the parts of your definition.

2. (10 pts.) Using your definition in part (1) give a definition of $\int_a^b f(x)\,dx$ for a function $f(x)$ on an interval $[a, b]$