General Notes:

1. **Show work.**
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam. Carry out any calculations to the point at which you would need a calculator (for example, to take the square root of the logarithm of a number) and leave it in that form.

Friday, Oct. 23, 2009
100 pts.
I. Limits and continuity

a. (5 pts.) Give a formal ($\varepsilon$ - $\delta$) definition of $\lim_{x \to a} f(x) = L$

b. (10 pts.) Explain as if to an intelligent friend who has not yet taken calculus what this means. Your explanation should involve more than translating symbols into words or explanations such as "as $x$ gets close to $a..."
(problem 1 continued)

c. (10 points) use the definition to show that $\lim_{x \to 1} (5x + 1) = 6$ by finding a suitable $\delta$ for $\varepsilon = 1/100$ and show that it works.
d. (5 pts.) Give a definition of the statement "f(x) is continuous at $x_0"$

e. (5 pts.) State the intermediate value theorem.

f. (5 pts.) Use the intermediate value theorem to show that $0 = x^2 - 2$ has a solution in the interval [0,2]
2. Differentiation

a. (5 pts.) Give the definition for the derivative of a function \( f(x) \) at the point \( x_0 \)

b. (10 pts.) Use the definition to find the derivative of \( f(x) = x^2 - 2 \)

From this point on in this exam, you can use the formulas we have developed. You do not need to use the limit definition from this point on.
3. Using the rules and formulas we have developed so far, calculate the derivatives of the following functions (5 pts. each): You do not need to carry out any polynomial multiplication or division.

a. \(3x^4 + 2x^3 + 5x + 17\)

b. \((3x^4 + 2x^3 + 5x + 17)(4x^3 + 12x - 2)\)

c. \(\frac{(3x^4 + 2x^3 + 5x + 17)}{(4x^3 + 12x - 2)}\)

d. \((3x^4 + 2x^3 + 5x + 17)^5\)
(problem 3 continued)

e. \( e^{\sin(2t)} \)

4. (10 pts.) Find the equation of the line tangent to \( y = 2x^2 - 8x \) at the point (1, -6)
5. (10 pts.) Find any vertical, horizontal, and oblique asymptotes of \( y = \frac{x^2 - 2}{x - 1} \). You do not need to graph the resulting function.