Math 180 F

SECOND HOUR EXAM

NAME__________________________________________

General Notes:

1. Show work.
2. Look over the test first, and then begin.
3. Calculators are not permitted on this exam.

Friday, Oct. 24, 2008
100 pts
I. Limits

1. (10 pts.) Give a formal \((\varepsilon - \delta)\) definition of \(\lim_{x \to a} f(x) = L\)

2. (10 pts.) Show that \(\lim_{x \to 1} (2x + 7) = 9\) by finding an appropriate \(\delta\) for \(\varepsilon = \frac{1}{10} (= 10^{-1})\). Be sure to show your work. Just writing down a \(\delta\) is not sufficient.
3. (10 pts.) Identify x and y intercepts (i.e., points at which the graph crosses the x and y axis) vertical, horizontal, and oblique asymptotes (if any) and give a brief sketch of the following function:

a. \[ y = \frac{x^2 + 4x + 4}{x - 1} \]

vertical:

horizontal:

oblique:

Sketch of graph
II. Continuity

1. (5 pts. each).

a. Define (formal definition) what it means for a function \( f \) to be continuous at a point \( x_0 \).

b. What is the intermediate value theorem for continuous functions?

c. Suppose that we know that a continuous function \( f(x) \) has \( f(1) = -3 \) and \( f(2) = 5 \). What can we say about a solution to the equation \( f(x) = 0 \)? Why?
II. Differentiation

1. (10 pts.) Give a formal definition of the derivative of a function $f(x)$ at a point $x_0$.

2. (10 pts.) Use the definition of the derivative to calculate $f'(x) = \frac{d}{dx} f(x)$ for $f(x) = x^2 + 2x + 1$
3. (5 pts each) In the following, calculate the derivative of the given function using the rules for calculating derivatives (i.e., you don't need to use the definition in these problems). Carry your answer to the point where there are no more

a. \( f(x) = 2x^3 + 3x^2 - 7x + 1 \)

b. \( f(x) = x^2 \sin(x) \)

c. \( f(x) = \frac{2x^2 + 1}{x^3 + 2x} \)

d. \( f(x) = \frac{1}{1 + e^{-x}} \)

e. \( f(x) = (2x^3 + 3x^2 - 7x + 1)^4 \)
4. (10 pts.) The graph of the curve $y = x^2 + 2x + 1$ passes through the point (1,4). Find the equation of the lines tangent to and normal to the curve at that point.