I affirm this work abides by the university’s Academic Honesty Policy.

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• First due date **Tuesday, February 18.**
• Turn in your work on a separate sheet of paper with this page stapled in front.
• Do not include scratch work in your submission.
• There is to be no collaboration on any aspect of developing and presenting your proof. Your only resources are: you, the course textbook, me, and pertinent discussions that occur during class.
• Follow the Writing Guidelines of the Grading Rubric in the course information sheet.
• Retry: Only use material from the relevant section of the text or earlier.
• Retry: Start over using a new sheet of paper.
• Retry: Restaple with new attempts first and this page on top.

“It is by logic that we prove but by intuition that we discover.” (Henri Poincaré)

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SLE-2 (Use only material up to and including Section HSE)
Let $A$ be an $m \times n$ matrix, and $\vec{b}$ a constant vector for which the system of equations $LS(A, \vec{b})$ is consistent and has solution set $S$. Pick one vector in $S$ and denote it by $\vec{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}$. Let $T$ be the set of all vectors obtained by adding the components of $\vec{\beta}$ to the corresponding components of each of the vectors in $N(A)$, the null space of $A$. More specifically, $T = \left\{ \begin{bmatrix} a_1 + \beta_1 \\ a_2 + \beta_2 \\ \vdots \\ a_n + \beta_n \end{bmatrix} \in \mathbb{C}^n \ \bigg| \ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \in N(A) \right\}$. Prove that the sets $S$ and $T$ are equal.
For all $m \times n$ matrices $A$ and all vectors $\vec{b} \in \mathbb{C}^m$ for which the system of equations $LS(A, \vec{b})$ is consistent with solution set $S$.

For all $\vec{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix} \in \mathbb{C}^n$,

prove, if $T = \begin{bmatrix} a_1 + \beta_1 \\ a_2 + \beta_2 \\ \vdots \\ a_n + \beta_n \end{bmatrix} \in \mathbb{C}^n \mid \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \in N(A) \} $, then $S = T$. 

For all $m \times n$ matrices $A$ and all vectors $\vec{b} \in \mathbb{C}^m$ for which the system of equations $LS(A, \vec{b})$ is consistent with solution set $S$.

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For all $m \times n$ matrices $A$ and all vectors $\vec{b} \in \mathbb{C}^m$ for which the system of equations $LS(A, \vec{b})$ is consistent with solution set $S$.

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