Definition 1  Let $W_1, W_2, \cdots$ be any collection of subsets of a set $V$. Define $\bigcap_{k=1}^{1} W_k = W_1$, and $\bigcap_{k=1}^{m+1} W_k = \left( \bigcap_{k=1}^{m} W_k \right) \cap (W_{m+1})$ for all integers $m \geq 1$.

1. Use the Principle of Mathematical Induction to prove the following theorem.

**Theorem 1**  If $W_1, W_2, \cdots, W_p$ are subspaces of a vector space $V$, then their intersection $\bigcap_{k=1}^{p} W_k$ is also a subspace of $V$.

2. Show that no analogous theorem can be true for unions by specifying two particular subspaces of $\mathbb{C}^3$ whose union is not a subspace of $\mathbb{C}^3$. Be sure to explain why the union is not a subspace.

3. Use the concept of dimension to determine all subspaces of $\mathbb{C}^3$. Then describe the geometric meaning of each type of subspace for vectors that have real numbers as entries.

Notes:

- The intersection of sets $S$ and $T$ is defined by $S \cap T = \{ x : x \in S \text{ and } x \in T \}$.
- The union of sets $S$ and $T$ is defined by $S \cup T = \{ x : x \in S \text{ or } x \in T \text{ (or both)} \}$.