V-2 (Section O) Prove both of the following Theorems.
The following two results (especially the first) might seem simple but they provide an excellent opportunity to learn how to correctly present a proof involving linear independence. So make sure to focus on the using correct notation to present the details.

Theorem 1 (Contract) Suppose \( n \geq 2 \) and that \( S = \{ \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{n-1}, \vec{v}_n \} \) is a linearly independent set of vectors. Then \( T = \{ \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{n-1} \} \) is also linearly independent.

Theorem 2 (Expand) Suppose \( S = \{ \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{n-1}, \vec{v}_n \} \) is a linearly independent set of vectors and that \( \vec{z} \notin \langle S \rangle \). Then \( W = \{ \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{n-1}, \vec{v}_n, \vec{z} \} \) is also linearly independent.

[These theorems are the keys to building larger (or smaller) linearly independent sets.]

"Know thyself? If I knew myself, I'd run away." – Johann von Goethe