Directions: Only write on one side of each page.

Do any (5) of the following

1. (20 points) Using any previous results, prove Proposition 4.7: Hilbert’s Euclidean parallel postulate \( \iff \) if a line intersects one of two parallel lines, then it also intersects the other.

2. (20 points) Using any previous results, prove the uniqueness (but not existence) part of Proposition 4.3: Every segment has a unique midpoint.

3. (20 points) Using any results through Chapter 5 prove that Hilbert’s Euclidean parallel property \( \iff \) Statement Ex5 where statement Ex5 is:
   Given lines \( l \) and \( m \) where \( l \parallel m \), point \( P \) is on \( m \), \( Q \) is the foot of the perpendicular from \( P \) to line \( l \), and \( R \) is the foot of the perpendicular from \( Q \) to line \( m \). Then \( \overline{PQ} = \overline{QR} \).

4. (20 points) Using any results through Chapter 4, prove the following: Hilbert’s parallel property holds \( \iff \) if \( k, m, l \) are distinct lines, \( k \) is parallel to \( m \), and \( m \) is parallel to \( l \), then \( k \) is parallel to \( l \).

5. (20 points) Using any results through Chapter 5, prove that Hilbert’s parallel postulate implies Wallis’ postulate. [Wallis’ postulate is: Given any triangle \( \triangle ABC \) and given any segment \( DE \). There exists a triangle \( \triangle DEF \) (having \( DE \) as one of its sides) that is similar to \( \triangle ABC \).]

6. (4 points each) Which of the following statements are correct? [You need not rewrite the statements when you answer.]
   
   (a) In hyperbolic geometry, if \( \triangle ABC \) and \( \triangle DEF \) are equilateral triangles and \( \angle A \cong \angle D \), then the triangles are congruent.

   (b) In hyperbolic geometry, if \( m \) contains a limiting parallel ray to \( l \), then \( l \) and \( m \) have a common perpendicular.

   (c) In hyperbolic geometry, if \( m \) does not contain a limiting parallel ray to \( l \) and if \( m \) and \( l \) have no common perpendicular, then \( m \) intersects \( l \).

   (d) Every valid theorem of neutral geometry is also valid in hyperbolic geometry.

   (e) In hyperbolic geometry, there exists an angle and there exists a line that lies entirely within the interior of this angle.