Extra Credit

• (2 points): What is the negation of the statement “For every line \( l \) and every line \( m \) not equal to \( l \), \( l \) and \( m \) are incident with exactly the same number of points”? You may use words, formal logical symbols, or a mixture of both.

Do any (5) of the following

1. (20 points) Give a detailed explanation of how and why we can use models to show that a statement \( S \) is independent of the axioms of an axiomatic system.

2. (10, 10 points) Given the following statement \( S \): “For every line \( l \) and every line \( m \) not equal to \( l \), \( l \) and \( m \) are incident with exactly the same number of points”.
   
   (a) Present a model of Incidence geometry that shows it is impossible, using the axioms of incidence geometry, to prove statement \( S \).
   
   (b) Present a model of Incidence geometry that shows it is impossible, using the axioms of incidence geometry, to prove the negation of statement \( S \).

3. (20 points) Using any results through the corollary to Betweenness Axiom 4, prove the Same Side Lemma: Given \( A \ast B \ast C \) and \( l \) and line other than line \( \overrightarrow{AB} \) meeting line \( \overrightarrow{AB} \) at point \( A \). Then \( B \) and \( C \) are on the same side of line \( l \).

4. (8, 8, 4 points) Show that it is possible for two four-point models of Incidence geometry to not be isomorphic by:
(a) Carefully stating what are the points, lines and incidence of both interpretations.

(b) Briefly illustrating why each is a model of Incidence geometry.

(c) Explaining how you know they are not isomorphic.

5. (20 points) Using any results from Incidence geometry, prove the following. In a finite affine plane in which every line has exactly 10 points then there cannot be more than 10 lines incident with any point. [Hint: start with an arbitrary point $P$ and Proposition 2.4 and recall that an affine plane is a model of incidence geometry in which the Euclidean parallel property holds.]

6. (20 points) Using any previous results, give a formal proof of Proposition 2.1: If $l$ and $m$ are distinct lines that are not parallel then $l$ and $m$ have a unique point in common.

7. (5, 15 points) Proposition 2.6 says: For every point $P$ there are at least two distinct points neither of which is $P$.

   (a) Restate this proposition in “If (hypothesis), then (conclusion)” form.

   (b) Using any previous results, give a formal proof of this proposition. [Be careful, there is nothing in the statement of the proposition that implies the point $P$ is one of the points guaranteed by Incidence Axiom 3.]