M-1 (Section MISLE)

1. Given an $m \times n$ matrix $A$ and an $n \times m$ matrix $B$ where $m > n$, prove that it is impossible to have the product $AB = I_m$.

2. Show that the order of multiplication is important by finding a specific $6 \times 4$ matrix $A$ and a specific $4 \times 6$ matrix $B$ where $BA = I_4$.

3. For bonus points, generalize your answer in part 2. That is, prove that for any positive integers $m$ and $n$ with $m > n$, then there is an $m \times n$ matrix $A$ and an $n \times m$ matrix $B$ with $BA = I_n$.

Notes:

- These matrices are not square so don’t use results that require square matrices.
- One way to approach part 1 is to think about null spaces.
- For parts 2 and 3, consider $\begin{bmatrix} 2 & 0 \\ \frac{1}{2} & 0 \end{bmatrix} = [1] = I_1$