I affirm this work abides by the university’s Academic Honesty Policy.

Print Name, then Sign

• First due date **Thursday, April 1**.
• Turn in your work on a separate sheet of paper with this page stapled in front.
• Do not include scratch work in your submission.
• There is to be **no collaboration** on any aspect of developing and presenting your proof. Your only resources are: you, the course textbook, me, and pertinent discussions that occur during class.
• Follow the Writing Guidelines of the Grading Rubric in the course information sheet.
• Retry: Only use material from the relevant section of the text or earlier.
• Retry: Start over using a new sheet of paper.
• Retry: Re-staple with new attempts first and this page on top.

“By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and, in effect, increases the mental power of the race.” – Alfred North Whitehead

**VS-1 (Section S)**

**Definition 1** Let $W_1, W_2, \cdots$ be any collection of subsets of a set $V$. Define $\bigcap_{k=1}^{0} W_k = \{ \}$, $\bigcap_{k=1}^{1} W_k = W_1$, and $\bigcap_{k=1}^{m+1} W_k = \left( \bigcap_{k=1}^{m} W_k \right) \cap (W_{m+1})$ for all integers $m \geq 1$.

1. Use the Principle of Mathematical Induction to prove the following theorem.

**Theorem 1** If $W_1, W_2, \cdots, W_p$ are subspaces of a vector space $V$, then their intersection $\bigcap_{k=1}^{p} W_k$ is also a subspace of $V$.

2. Show that no analogous theorem can be true for unions by specifying two particular subspaces of $\mathbb{C}^3$ whose union is not a subspace of $\mathbb{C}^3$. Be sure to explain why the union is not a subspace.

**Notes:**

• The intersection of sets $S$ and $T$ is defined by $S \cap T = \{ x : x \in S \text{ and } x \in T \}$.
• The union of sets $S$ and $T$ is defined by $S \cup T = \{x : x \in S \text{ or } x \in T \text{ (or both)}\}$

• When considering specific subspaces, the easiest ones to look at are those that are the spans of sets of vectors.