M-2 (Section FS)

Definition: If $A$ is a square matrix of size $m$ then we define $A^0 = I_m$, $A^1 = A$, and $A^{n+1} = A^n A$ for each $n \geq 1$. Further, if $A$ is invertible, we define $A^{-n} = (A^{-1})^n$.

1. Suppose $A$ and $B$ are square matrices of size $m$ and that $A$ is non-singular. Use the principle of mathematical induction to prove that $(A^{-1}BA)^n = A^{-1}B^nA$ for every positive integer $n$.

2. Now suppose that $B$ is also nonsingular and extend the previous result by proving the formula $(A^{-1}BA)^n = A^{-1}B^nA$ holds for every integer (positive, negative and zero).

3. Use your formula and the matrices $B = \begin{bmatrix} \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$ and the vector $\vec{x}_0 = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$ to compute $B^n\vec{x}_0$. What is the component by component limit of $B^n\vec{x}_0$ as $n \to \infty$?

Notes:

- In part 3, $A^{-1}BA$ should simplify to be a diagonal matrix.

- Recall the formula for powers of diagonal matrices (proven in class) and use it to compute $B^n$. 

"True eloquence consists in saying all that is necessary, and nothing but what is necessary." – La Rochefoucauld