Math 434 Midterm: Topic Review

Here is a list of the major topics we have covered so far this semester.

Rubik’s Cube
I will not ask any specific questions about Rubik’s cube.

Group Theory
- Generalized Cayley Theorem
  1. Index theorem (corollary) applications to existence of normal subgroups.
- Counting
  1. Burnside’s Theorem
- Proofs of Group Isomorphism Theorems (these get a heavy workout later)
  1. First, Second, Third Isomorphism Theorems
  2. Correspondence Theorem
- Classification of Finite Abelian Groups.
  1. Still not completely proven
  2. Should be able to deduce and use the possible abelian groups of any order
    (a) For a simple example: $|G| = 2^3 \cdot 7$ and $G$ abelian implies $G$ is one of the following:
      i. $Z_2 \times Z_2 \times Z_2 \times Z_7$
      ii. $Z_4 \times Z_2 \times Z_7$
      iii. $Z_8 \times Z_7$

Ring Theory
- Peano Axioms about $Z$
- Homomorphisms and Ideals
- Quotient Rings
- Adjunction of Elements
- Fraction Fields obtained from Integral Domains
- Link between Maximal ideals and Fields
- Factoring in:
1. $\mathbf{Z}, \mathbf{F}[x], \mathbf{Z}[i]$
2. Unique Factorization Domains
3. Principle Ideal Domains
4. Euclidean Domains