I affirm this work abides by the university’s Academic Honesty Policy.

Print Name, then Sign

Directions:

- Only write on one side of each page.
- Use terminology correctly.
- Partial credit is awarded for correct approaches so justify your steps.

Define all three of the following.

D.1. [6 points] The coordinate transformation $\rho_B$ where $B = \{b_1, \cdots, b_n\}$ is a basis for the vector space $V$.

D.2. [7 points] A linearly dependent subset $S$ of a vector space $V$.


Do one (1) of these "Computational" problems

C.1. [15 points] Given a vector space $W$ and two subsets of $W$: $S = \{\vec{w}_1, \vec{w}_2, \vec{w}_3, \vec{w}_4\}$ and $T = \{\vec{w}_1 + 2\vec{w}_2 + 3\vec{w}_3 + 4\vec{w}_4, 2\vec{w}_1 - \vec{w}_2 + 5\vec{w}_3 - 3\vec{w}_4, \vec{w}_1 + 5\vec{w}_2 + 6\vec{w}_3, -3\vec{w}_1 + 2\vec{w}_2 + \vec{w}_3 + 2\vec{w}_4\}$. If $S$ is linearly independent in $W$ either prove that $T$ is also linearly independent or write one of the four vectors in $T$ as equal to a linear combination of the other three vectors in $T$. Show all work.

C.2. [15 points] Given the linear transformation $T : M_{22} \to M_{22}$ given by $T(A) = A + A^t$. Find the matrix representation $M^T_{B,C}$, where $B$ is the standard basis of $M_{22}$ and $C = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 Points] A certain $5 \times 5$ matrix $C$ can be written as $C = AB$ where $A$ is $5 \times 4$ and $B$ is $4 \times 5$. Explain how you know that $\det(C) = 0$.

M.2. [15 Points] Prove Theorem AIU from our textbook.

**Theorem AIU:** Suppose that $V$ is a vector space. For each $\vec{u} \in V$, the additive inverse, $-\vec{u}$ is unique. (You may not use the fact that $-\vec{u} = (-1)\vec{u}$.)

M.3. [15 Points] Prove Theorem CILTI from the textbook.

**Theorem CILTI:** Suppose that $T : U \to V$ and $S : V \to W$ are both injective linear transformations. Then $(S \circ T) : U \to W$ is an injective linear transformation. You may use, without proving it, the fact that $S \circ T$ is a linear transformation.
Do three (3) of these "Other" problems

T.1. [15 Points] Recall the definition given in class that a square matrix $A$ of size $n$ is **skew-symmetric** if $A^t = -A$. Prove that if $n$ is an odd integer then $A$ is not invertible. [Hint: Consider determinants.]

T.2. [15 Points] Suppose $T : U \rightarrow V$ is a function that satisfies the single condition $T(\alpha \vec{x} + \vec{y}) = \alpha T(\vec{x}) + T(\vec{y})$ for every $\vec{x}, \vec{y}$ in $U$ and every $\alpha$ in $C$. Prove that $T(\vec{0}) = \vec{0}$. You may not use the fact that $T$ is a linear transformation.

T.3. [15 Points] Prove that the matrices $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are similar by finding a matrix $S$ for which $A = S^{-1}BS$.

T.4. [15 points] Let $B = \{e^x, xe^x, x^2 e^x\}$ be a basis for the subspace $V$ of the vector space $F$ of functions with domain and codomain the set of complex numbers: $F = \{ f : C \rightarrow C \}$.

1. Find the matrix representation $M^T_{B,B}$ of the linear transformation $T : V \rightarrow V$ defined by $T(f) = f'$.
2. Use this matrix representation to find the kernel of $T$, ker $(T)$.

You must do both of these problems ON THIS SHEET

R.1. [15 points] Prove that the set $V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{C}^3 : 2x_1 - 7x_2 + x_3 = 0 \right\}$ is a subspace of $\mathbb{C}^3$ by applying the three-part test of Theorem TSS. Write your proof according to the standards of this semester’s writing exercises.

R.2. [15 points] Part of Theorem NPNT (“Nonsingular Products, Nonsingular Terms”) says: If $A$ and $B$ are square matrices of the same size, and $AB$ is nonsingular, then $B$ is nonsingular. Construct a proof by contradiction of this fact and write your proof according to the standards of this semester’s writing exercises. (You may not do this problem by simply quoting Theorem NPNT.)