Do BOTH of these “Computational” Problems

C.1. Solve the following system of linear equations by hand. Write the solution set using column vector notation. Make sure you copy the equations correctly.

\[
\begin{align*}
    x_4 + 2x_5 - x_6 &= 2 \\
    x_1 + 2x_2 + x_5 - x_6 &= 0 \\
    x_1 + 2x_2 + 2x_3 - x_5 + x_6 &= 2 \\
    x_1 + 2x_2 + 2x_3 + x_4 + x_5 &= 4
\end{align*}
\]

C.2. Find a 4 \times 5 matrix \( A \), that is not in reduced row-echelon form, whose null space is the set

\[
\left\{ \begin{bmatrix}
2x_2 - 6x_4 \\
x_2 \\
-5x_4 \\
x_4 \\
7x_2 + x_4
\end{bmatrix} : x_2, x_4 \in \mathbb{C} \right\}
\]

Do Two (2) of these “In text, class or homework” problems

M.1. Suppose \( A \) and \( B \) are \( m \times n \) matrices. Give a detailed explanation of why if \( A \) is row-equivalent to \( B \) then \( B \) is row-equivalent to \( A \).

M.2. Suppose that \( B \) is an \( m \times n \) matrix in reduced row-echelon form. Build a new, likely smaller, \( k \times l \) matrix \( C \) as follows. Keep any collection of \( k \) adjacent rows, \( k \leq m \). From these rows, keep columns 1 through \( l \), \( l \leq n \). Prove that \( C \) is in reduced row-echelon form.

M.3. Prove that a system of linear equations is homogeneous if and only if it has the zero vector as a solution.
Do BOTH of these “Not in Text” problems

T.1. Suppose that \( A \) is the coefficient matrix of a consistent linear system of equations and that two of the columns of \( A \) are identical. Prove that there must be an infinite number of solutions to the system of equations.

T.2. Let \( A \) be a \( 4 \times 4 \) matrix and let \( \vec{b} \) and \( \vec{c} \) be vectors of constants with 4 entries each.

(a) If we are told that the linear system of equations \( LS\left( A, \vec{b} \right) \) has a unique solution, what can you say about the solutions set of \( LS\left( A, \vec{c} \right) \)? Why?

(b) If we are told that the linear system of equations \( LS\left( A, \vec{b} \right) \) is inconsistent, what can you say about the solutions set of \( LS\left( A, \vec{c} \right) \)? Why?

(c) Now suppose \( B \) is a \( 4 \times 3 \) matrix and we know that \( LS\left( B, \vec{b} \right) \) has a unique solution. What can you say about the solution set of \( LS\left( B, \vec{c} \right) \)? Why?