Complete the following definitions

D.1. [3 points] Given a finite set of vectors \( S \subset \mathbb{C}^m \), the span of \( S \) is

D.2. [3 points] The null space of a matrix \( A \), denoted \( N(A) \) is

D.3. [4 points] A matrix \( A \) is in reduced row-echelon form if it meets all of the following conditions

Do both of these "Computational" problems

C.1. [10, 5 points] Solve the following system of linear equations by hand. Write the solution set using column vector notation.

\[
\begin{align*}
  x + 3y + 4w &= 3 \\
  -z - 5w &= -2 \\
  x + 3y + z + 9w &= 5 \\
  2x + 6y + 8w &= 6
\end{align*}
\]

C.2. [15 points] Suppose we run the sequence of three row operations:

\[
\begin{align*}
  [A \mid \vec{b}] & \xrightarrow{R_2 \leftrightarrow R_4} [B \mid \vec{c}] & \xrightarrow{2R_3} & [C \mid \vec{d}]
\end{align*}
\]

where \([D \mid \vec{e}] = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 0 \\ 1 & -1 & 2 \\ 0 & 1 & 1 \end{bmatrix}\). What is \( C \) if \( C \) is the result of running the row operation \(-4R_2 + R_1\) on \( A \)?

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Is there a parabola whose graph passes through the points \((1, 3)\), \((2, 6)\), \((4, -1)\), \((5, 0)\)? Give careful reasons for your answer.

M.2. [15 points] Property SMAC of column vectors is: if \( \alpha, \beta \in \mathbb{C} \) and \( u \in \mathbb{C}^m \), then \( \alpha (\beta u) = (\alpha \beta) u \). Prove this property and write your proof in the style of the proof of Property DSAC given in the textbook.

M.3. [15 points] Prove that a system of linear equations is homogeneous if and only if it has the zero vector as a solution.
Do any two (2) of these ”Other” problems

T.1. [10, 5 points] Suppose that $\alpha$ is any constant and that $\mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$ is a solution of the homogeneous system of linear equations $LS(A, \mathbf{0})$. Prove that $\mathbf{t} = \begin{bmatrix} \alpha u_1 \\ \vdots \\ \alpha u_n \end{bmatrix}$ is also a solution of $LS(A, \mathbf{0})$. [Be sure to explicitly show that $\mathbf{t}$ solves the system of equations.]

T.2. [15 points] Consider the system of $m$ linear equations in $n$ variables $LS(A, \mathbf{b})$, and suppose that the vector $\mathbf{b}$ equals twice one column vector of $A$ plus five times a different column vector of $A$. More precisely, there are two column indices $j_1$ and $j_2$ such that for each $i, 1 \leq i \leq m$, it is the case that $[\mathbf{b}]_i = 2[A]_{i,j_1} + 5[A]_{i,j_2}$. Prove that the system is consistent.

T.3. [15 points] Let $S = \{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ and $T = \{\vec{w}_1, \vec{w}_2, \vec{w}_3, \vec{w}_1 + 2\vec{w}_2 + 3\vec{w}_3\}$ be subsets of $\mathbb{C}^m$. Prove that $\langle S \rangle = \langle T \rangle$. 

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