Do six (6) of the following problems

1. (8, 7 points) Given the function $f(x, y, z) = \ln \left( \frac{x^2 + y^2}{25} + \frac{z}{9} \right)$

   (a) Find an equation for the level surface of $f$ that passes through the point $(5\sqrt{2}, 4, 9)$.

   (b) Draw a reasonably careful sketch of that level surface.

2. (3, 12 points) Suppose $f(x, y) = \frac{9x^2 - y^2}{x^2 + 4y^2}$ for all $(x, y) \neq (0, 0)$.

   (a) Is there a number $k$ that makes the function $g$ given below continuous at $(0, 0)$?

   $$g(x, y) = \begin{cases} f(x, y) & \text{if } (x, y) \neq (0, 0) \\ k & \text{if } (x, y) = (0, 0) \end{cases}$$

   (b) Why or why not?

3. Reparametrize the following curve by arclength. That is, express this curve in terms of the arclength parameter $s(t) = \int_0^t \| \vec{r}'(\tau) \| \, d\tau$.

   $$\vec{r}(t) = (\cos t + t \sin(t)) \hat{i} + (\sin t - t \cos t) \hat{j}, \quad \frac{\pi}{2} \leq t \leq \pi$$

4. (5, 5 points) Find $T, N$ and the curvature $\kappa$ for the parametrized space curve $\vec{r}(t) = (\cos^3 t) \hat{i} + (\sin^3 t) \hat{j}$, $0 < t < \pi/2$.

5. If $w = \sin(2x + y)$, $x = \sin(\pi s)$, and $y = rs$, find the value of the following second partial derivative at the point where $r = \pi$ and $s = 1$.

   $$\frac{\partial^2 w}{\partial r \partial s}$$

6. Write a chain rule formula for $\frac{\partial w}{\partial t}$ if $w = f(x, y)$, $x = g(t, s)$, and $y = h(t, s, x)$ are all differentiable functions of their respective input variables.
7. The space curve \( \mathbf{r}(t) = \left( \frac{1}{3}t^3 \right) \hat{i} + (t^2) \hat{j} + (2t) \hat{k} \) has unit tangent vector \( \mathbf{T}(t) = \frac{t^2}{t^2 + 2} \hat{i} + \frac{2t}{t^2 + 2} \hat{j} + \frac{2}{t^2 + 2} \hat{k} \) and taking the derivative we find that
\[
\frac{d\mathbf{T}}{dt} = \left( \frac{4t}{(t^2 + 2)^2} \right) \hat{i} + \left( \frac{-2t^2 + 4}{(t^2 + 2)^2} \right) \hat{j} + \left( \frac{-4t}{(t^2 + 2)^2} \right) \hat{k}
\]

(a) What is an equation of the osculating plane for this curve at the point \( x = 2 \)?

(b) What are the center and radius of the osculating circle (also known as the circle of curvature) for this curve at the point where \( t = 2 \)?

8. We showed in class that if \( \mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle \) is a smooth parametrized space curve satisfying \( \| \mathbf{r}(t) \| = c \) for every \( t \) in the domain of \( \mathbf{r} \) then \( \mathbf{r}(t) \) and \( \mathbf{r}'(t) \) are perpendicular vectors for every \( t \) in that domain. By using components and integration, show that the converse is also true by proving the following.

**Theorem** If \( \mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle \) is a smooth parametrized space curve in which \( \mathbf{r}(t) \cdot \mathbf{r}'(t) = 0 \) for every \( t \) in the domain of \( \mathbf{r} \), then there is a constant \( c \) for which \( \| \mathbf{r}(t) \| = c \) for every \( t \) in the domain of \( \mathbf{r} \).