You MUST do A. and one part of B.

A. [10 points] Reverse the order of integration in the following double integral Do Not evaluate the integral.

\[
\int_{0}^{3} \int_{\sqrt{y/3}}^{1} e^{(y^2)} \, dy \, dx.
\]

B. Do one (1) of the following:

(a) [7, 8 points] Find and classify all local maxima, local minima and saddle points of the function \( f(x, y) = x^4 + y^4 + 4xy \).

(b) [15 points] Find the absolute minimum value of the function \( f(x, y) = 48xy - 32x^3 - 24y^2 \) on the rectangular plate \( 0 \leq x \leq 1, \, 0 \leq y \leq 1 \).

Do any FIVE (5) of the following

1. [10, 5 points] Given \( f(x, y) = 49 - x^2 - y^2 \)

   (a) If possible, maximize \( f(x, y) \) subject to the constraint \( x + 3y = 10 \).

   (b) Explain why or why not this constrained optimization has an absolute maximum.

2. [15 points] The area charge density function for a region in the \( xy \)-plane bounded by the cardioid \( r = 1 + \sin(\theta) \) is proportional to the square of the distance from the origin with the maximum \( \sigma_0 \) occurring at the point with polar coordinates \( [2, \pi/2] \). Express the total charge as an iterated double integral in polar coordinates. Do Not evaluate your integral.

3. [15 points] It can be shown that the improper integral \( I = \int_{0}^{\infty} e^{-x^2} \, dx \) converges. The usual way to determine the value is to first calculate its square

\[
I^2 = \left( \int_{0}^{\infty} e^{-x^2} \, dx \right) \left( \int_{0}^{\infty} e^{-y^2} \, dy \right) = \int_{0}^{\infty} \int_{0}^{\infty} e^{-(x^2+y^2)} \, dx \, dy.
\]

Evaluate the last integral using polar coordinates and solve the resulting equation for \( I \).

4. [15 points] Change the order of integration to the order \( dz \, dx \, dy \), but do not evaluate, the following triple integral.

\[
\int_{0}^{1} \int_{0}^{1-x^2} \int_{0}^{x} \frac{\sin(2z)}{4-z} \, dy \, dz \, dx.
\]
5. [15 points] Each point of the portion of the solid sphere $\rho \leq a$ that lies between the cone $\phi = \frac{\pi}{3}$ and the plane $z = 0$ has a volume charge density proportional to the distance of the point from the origin. The maximum volume charge density of $\delta_0$ occurs along the circle where the cone meets the sphere. Find the total charge on the solid.

6. [15 points] Let $n$ be a positive integer. Set up and evaluate a definite integral that gives the length of a helix that wraps 17 times around the lateral side of a right circular cylinder of radius $R$ and height $H$ with a constant pitch (so each wrap rises the same distance up the cylinder). Your answer should not have any integral signs and will involve the letters $R$ and $H$. 