Technology used: Only write on one side of each page.

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

“A chief event of life is the day in which we have encountered a mind that startled us”. -Ralph Waldo Emerson, writer and philosopher (1803-1882)

Problems

1. [15 points] From the quiz: Find a polynomial that will approximate the following function throughout the interval [0, 0.5] with an error of magnitude less than $10^{-3}$

$$F(x) = \int_0^x \arctan(t) \, dt$$

2. [15 points] Using any results from Chapter 8, build the Maclaurin Series for the following function. [Exploiting known Taylor Series is the easiest approach.]

$$f(x) = \frac{x^3}{1 - 3x}$$

3. [10 points each] Do any two (2) of the following.

Use appropriate convergence tests to determine whether the following series of positive terms converge or diverge. Show your work.

(a) \( \sum_{n=1}^{\infty} \frac{1}{2\sqrt{n} + \sqrt{n}} \)

(b) \( \sum_{n=1}^{\infty} \left(1 - \frac{1/3}{n}\right)^n \)

(c) \( \sum_{n=1}^{\infty} \frac{(10,000)^n}{n!} \)

(d) \( \sum_{n=1}^{\infty} \frac{1}{1 + 2 + 3 + \cdots + n} \)

4. [15 points] Use an appropriate convergence test to determine if the following series of positive terms converges

$$\sum_{n=0}^{\infty} \frac{n! (n + 1)! (n + 2)!}{(3n)!}$$

5. [15 points] Determine the radius of convergence, interval of convergence and numbers where convergence is conditional for the power series

$$\sum_{n=0}^{\infty} \frac{(4x - 5)^{n+1}}{2n}$$

6. [10 points] Prove the theorem that absolute convergence implies convergence. More specifically, prove that if the series \( \sum_{n=1}^{\infty} |a_n| \) converges then so does the series \( \sum_{n=1}^{\infty} a_n \).
7. [10 points] Replace the following equation with an equivalent polar equation.

\[(x + 2)^2 + (y - 5)^2 = 16\]

8. **Extra Credit.** [5 points] Give an example of converging series \(\sum_{n=1}^{\infty} a_n\) and \(\sum_{n=1}^{\infty} b_n\) for which the series \(\sum_{n=1}^{\infty} a_n b_n\) diverges. [Note that the terms do not need to be positive.]

**Useful Information**

- The Taylor series generated by the function \(f\) at \(x = a\) is

\[
\sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(a) \ (x - a)^n
\]

and the remainder term is

\[
R_n(x) = \frac{1}{(n+1)!} f^{(n+1)}(c) \ (x - a)^{n+1}
\]

where \(c\) is a number between \(a\) and \(x\).

- If \(\sum_{n=0}^{\infty} (-1)^n u_n\) is a convergent alternating series with sum \(S\) and if

\[
s_n = \sum_{k=0}^{n} (-1)^k u_k = u_0 - u_1 + u_2 - u_3 + \cdots + (-1)^n u_n
\]

is the \(n\)'th partial sum of the original series, then the \(n\)'th partial sum approximates the exact sum to within \(u_{n+1}\). That is, \(|S - s_n| < u_{n+1}\).

- The Maclaurin series for the arctangent function is

\[
\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n + 1}, \quad |x| \leq 1.
\]