Do this problem

1. Set up, but do not evaluate, definite integrals for two (2) of the following.

   (a) An integral representing the volume of the solid with base the region in the \(xy\)-plane bounded by the \(x\)-axis and the graph of the curve \(y = \sqrt{16 - x^2}\) and with cross sections perpendicular to the \(x\)-axis that are semicircles with diameter lying in the \(xy\)-plane.

   (b) An integral representing the volume of the solid or revolution obtained by revolving the region bounded by the graphs of \(y = 4 - x^2\) and \(y = 2 - x\) about the line \(y = -2\).

   (c) An integral representing the volume of the solid or revolution obtained by revolving the region bounded by the graphs of \(y = 4 - x^2\) and \(y = 2 - x\) about the line \(x = 3\).

Do any four (4) of the following problems

1. Find the area of the region bounded by the graph of \(y = \frac{\pi}{2} \cos (x) [\sin (\pi + \pi \sin (x))]\) over the interval \(-\pi \leq x \leq -\frac{\pi}{2}\). Note that \(y = 0\) at \(x = -\pi\) and \(x = -\pi/2\).

2. Find the length of the curve given by the parametrization \(x = \frac{1}{3} (2t + 3)^{3/2}, \ y = t + \frac{1}{2}t^2, \ 0 \leq t \leq 3\).

3. Suppose that a cup of soup cooled from 90° C to 60° C after 10 minutes in a room whose temperature was 20° C. Use Newton’s law of cooling to determine how much longer it would take for the soup to cool to 35° C.

4. A certain population of sheep (where \(y\) is measured in thousands of sheep) is modeled by the differential equation \(\frac{dy}{dt} = 2y (1 - y)\). Show that the function displayed below solves this separable differential equation. [Note that you are not being asked to solve the differential equation.]

   \[ y(t) = \frac{1}{1 + e^{-2t}} \]
5. For this problem pretend that you know the formula for the area inside a circle of radius $R$ is $\pi R^2$ but that you do not know the formula for the circumference of a circle. Let $C(x)$ be the notation for the function that gives the circumference of a circle of radius $x$.

Draw a picture in which you partition the interval $0 \leq x \leq R$ and then use your understanding of Riemann sums to build a definite integral, with integrand involving $C(x)$, that equals the area inside a circle of radius $R$. Briefly explain your integral formula but do not evaluate this integral.