The Problems

1. (20 points) On the attached sheet of graph paper, sketch the graph of the function \( f \) that satisfies the following conditions.

   Points on the graph of \( f \)  \((-4, -1), (-3, 1), (-1, -2), (0, 0), (3, 2), (4, 4)\)

   Inputs where \( f'(x) \) Does Not Exist  \( x = -4, x = -1 \)

   Inputs where \( f'(x) = 0 \)  \( x = -3, x = 3 \)

   Intervals where \( f'(x) > 0 \)  \( (-\infty, -3), (-1, 3) \)

   Intervals where \( f'(x) < 0 \)  \( (-3, -1) \)

   Intervals where \( f''(x) > 0 \)  \( (-\infty, -4), (3, 4) \)

   Intervals where \( f''(x) < 0 \)  \( (-4, -1), (-1, 3), (4, \infty) \)

   Limit information  \( \lim_{x \to -4} f'(x) = \infty \)

   \( \lim_{x \to -1^-} f'(x) = -\infty \)

   \( \lim_{x \to -1^+} f'(x) = 2 \)

2. (15 points) Find the absolute maximizers, minimizers, maximum and minimum of \( f(x) = x^{2/3} \) on \([-1, 8]\) or show they do not exist.

3. Given the function \( f(x) = -x^3 + 3x^2 - 1 \).

   (a) (10 points) Find the absolute maximum of \( f \) on \([0, \infty)\).

   (b) (10 points) Use calculus to carefully explain why this is an absolute maximum.

4. (20 points) Given the function \( f(x) = (x^2 - 1)^2 \).

   (a) Find all critical points of \( f \).

   (b) Find all second order critical points of \( f \).

   (c) Determine all intervals where \( f \) is strictly increasing and all intervals where \( f \) is strictly decreasing.

   (d) Determine all intervals where \( f \) is concave up and all intervals where \( f \) is concave down.

   (e) Classify the critical points as local maximizers, local minimizers or neither.

5. (15 points) Use Rolle’s Theorem or the Mean Value Theorem to show that if \( f \) is a polynomial with at least three zeros, say \( f(x_1) = f(x_2) = f(x_3) = 0 \), then there must be a point \( c \) at which \( f''(c) = 0 \).