Estimating greatest rate of change

The accompanying plot shows level curves for a function $f : \mathbb{R}^2 \to \mathbb{R}$. We can think of each input as a point on a plane and the corresponding output as a temperature. We will consider distance to be measured in kilometers (km) and temperature to be measured in degrees Celsius ($^\circ$C). There is a scale for distance at the bottom of the plot. A selection of level curves is labeled with the corresponding temperature.

1. For the point $A$, estimate the direction of the greatest rate of change in temperature with respect to change in position.

2. For the point $A$, estimate the magnitude of this greatest rate of change.

3. Choose a scale for rate of change. Note that this scale is independent of the scale for distance. With the temperature interpretation, rate of change has units of degrees Celsius per kilometer ($^\circ$C/km) while the length scale is in kilometers (km). To choose a scale for rate of change, go to the bottom of the plot next to the given length scale and draw a horizontal vector (of any size you want) to represent a magnitude of 1 $^\circ$C/km. You will use this choice in what follows.

4. At the point $A$, draw a vector in the direction of the greatest rate of change having magnitude equal to that rate of change. Use the rate of change scale you chose in #3.

5. For the point $B$, estimate the direction of the greatest rate of change in temperature with respect to changes in position.

6. For the point $B$, estimate the magnitude of this greatest rate of change.

7. At the point $B$, draw a vector in the direction of the greatest rate of change having magnitude equal to that rate of change. Use the rate of change scale you chose in #3.
Level curves for temperature as a function of position.
A
1 kilometer
T = 4.2
T = 4.8
T = 5.4
T = 6.0
T = 6.6
T = 7.2
T = 7.8

Zooming in on Point A
A

Zooming in on Point A
Zooming in on Point A
Zooming in on Point A
Zooming in on Point B
Zooming in on Point $B$

$T = -8.42$

$T = -8.28$

$T = -8.14$

$T = -8.00$

$T = -7.86$

$T = -7.72$

$T = -7.58$
Zooming in on Point B

0.01 kilometer
Zooming in on Point B
Level curves for temperature as a function of position.
Level curves with the vectors of Steps #4 and #7 included. Note that a scale for rate of change is also included.
Greatest rate of change vectors for a variety of points. Note that the scale for rate of change in this plot differs from the scale in the previous plot.