Definition 1 If \( A \) is an \( n \times p \) matrix and \( \vec{b} \) is a vector in \( \mathbb{C}^n \) then the matrix \( M_k \) is the matrix obtained by replacing the \( k \)th column of \( A \) with the vector \( \vec{b} \).

Prove the following theorem.

Theorem 1 If \( A \) is a nonsingular matrix of size \( n \) then the unique solution to the system of equations \( A\vec{x} = \vec{b} \) is the vector \( \vec{x} \) whose \( k \)th component is \( [\vec{x}]_k = \frac{\det(M_k)}{\det(A)} \).

**Hint:** Consider the matrix \( X_k \) obtained by replacing the \( k \)th column of the identity matrix with the vector \( \vec{x} \).