These and all previous problems MUST be turned in before October 13

“To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty of nature. If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in”. -Richard Feynman (1918-1988)

Problems

1. Do the following.

   (a) Prove that the relation $x$ is conjugate to $y$ in a group $G$ is an equivalence relation on $G$. (Definition: If $x$ and $y$ are in a group $G$ then we say $x$ is conjugate to $y$ in $G$ if there is an element $g$ in $G$ for which $x = gyyg^{-1}$)

   (b) Describe the elements $a$ whose conjugacy class (that is, whose equivalence class $[a]$) consists of the element $a$ alone.

   (c) Prove that the set $Z = \{a \in G : [a] = \{a\}\}$ is a subgroup of $G$. What is the name of this subgroup? (You have seen this subgroup before.)

2. Classify all groups of order less than or equal to 6. Use Lagranges theorem to divide your analysis into cases as we did in class when we classified the groups of order 4.