Problems

1. Do all of the following.
   
   (a) #11, 14, 21, 22b, 22d of Judson Chapter 1 (Pages 16,17)

2. Let \( f : X \rightarrow Y \) be a map of sets with \( A_1, A_2 \subset X \) and \( B_1, B_2 \subset Y \).

   Do all of the following
   
   (a) Prove \( f (A_1 \cap A_2) \subset f (A_1) \cap f (A_2) \) and give an example where equality fails.
   (b) Prove \( f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2) \)
   (c) Prove \( f^{-1}(Y \setminus B_1) = X \setminus f^{-1}(B_1) \)
   (d) Look at problem 24 of Chapter 1 of Judson and note that, in general, inverse images behave
      nicely’ with respect to intersections but not so nicely under unions.

3. Let \( f : A \rightarrow B \) be a surjective map of sets.

   (a) Prove that the relation defined by \( a \sim b \) if and only if \( f(a) = f(b) \) is an equivalence relation.
   (b) Prove that the equivalence classes of this relation are the fibers of \( f \).