1. [10 points] Do one (1) of the following.

1. A point P in the first quadrant lies on the graph of the function \( f(x) = \sqrt[3]{x} \). Express the \( x \)-coordinate of \( P \) as a function of the slope of the line joining \( P \) to the origin.

   **Solution:** The slope from the point \( P(x, \sqrt[3]{x}) \) to the origin is \( m = \frac{\sqrt[3]{x} - 0}{x - 0} = \frac{1}{x^{2/3}} = x^{-2/3} \). This means that \( x = \left(x^{-2/3}\right)^{-3/2} = m^{-3/2} \) Thus \( x = f(m) = m^{-3/2} \).

2. If a composite \( f \circ g \) is one-to-one. Then there are numbers \( x_1, x_2 \) with \( x_1 \neq x_2 \) for which \( g(x_1) = g(x_2) \). Thus, \( f(g(x_1)) = f(g(x_2)) = (f \circ g)(x_2) \). But this can’t happen because we are told that \( f \circ g \) is one-to-one. We can therefore deduce that \( g \) can’t be one-to-one.

2. [15 points] Rewrite the following sum as indicated.

   **Solution:** We make the change of index \( j = k + 11 \) which tells us that \( k = j - 11 \) then filling in the missing information we get

   \[
   \sum_{k=4}^{101} (2k - 1)^2 = \sum_{j=15}^{101+11} (2(j - 11) - 1)^2 = \sum_{j=15}^{112} (2j - 23)^2
   \]

3. [15 points] Do one (1) of the following. Show your work.

   1. Evaluate \( \int \frac{1}{t^2 - 3t^5 + t^{1/2} + 5t^3 \sec(t) + 6t^3 \sec(t) \tan(t) + \frac{t^3}{\sqrt[3]{1-t^2}}} \) \( \mathrm{d}t \)

      **Solution:** Multiplying through by \( \frac{1}{t^2} \) and using standard antiderivative formulas, we get

      \[
      \int \left( \frac{1}{t} - 3t^2 + t^{-5/2} + 5 \sec(t) + 6 \sec(t) \tan(t) + \frac{1}{\sqrt[3]{1-t^2}} \right) \mathrm{d}t = \ln|t| - t^3 + t^{-3/2} + 5 \tan(t)
      \]

   2. By differentiating the right hand side, verify the formula \( \int \frac{\arctan(x)}{x^2} \) \( \mathrm{d}x \) = \( \ln(x) - \frac{1}{2} \ln(1 + x^2) - \frac{\arctan(x)}{x} + C \)

      **Solution:**

      \[
      \frac{d}{dx} \left[ \ln(x) - \frac{1}{2} \ln(1 + x^2) - \frac{\arctan(x)}{x} + C \right] = \frac{1}{x} - \frac{1}{2} \cdot \frac{1}{1+x^2} (2x) - \frac{\frac{1}{1+x^2}(x)-(1)\arctan(x)}{x^2} = \frac{x^2 + 1}{x(x^2 + 1)} - \frac{\arctan(x)}{x^2} = 0 + \frac{\arctan(x)}{x^2}.
      \]
4. [8, 7 points] The following is a Riemann sum for a function \( f \) with domain an interval \([a, b]\). [Do NOT simplify this sum.]

\[
\sum_{k=1}^{n} \left[ 3 \left(5 + \frac{6k}{n}\right)^7 - 7 \left(5 + \frac{6k}{n}\right)^2 + 6 \right] \frac{6}{n}
\]

1. What is this specific \( f(x) \)?
2. What is the specific interval \([a, b]\)?

**Solution:** This Riemann sum has the form \( \sum_{k=1}^{n} f(c_k) \Delta x \) which tells us that \( \Delta x = \frac{6}{n} \) so we know that whatever \([a, b]\) is, we must have \( b - a = 6 \). We also see that the \( \frac{6k}{n} \) terms look like \( k \Delta x \) so we deduce that \( c_k = 5 + k \Delta x = 5 + \frac{6k}{n} \) this also tells us that our partition starts at \( a = 5 \) and contains \( 5 + 1 \Delta x, 5 + 2 \Delta x, \) etc. Thus we have \( f(x) = 3x^7 - x^2 + 6 \) with \([a, b] = [5, 11]\). An equally valid answer is \( f(x) = 3(5 + x)^7 - (5 + x)^2 + 6 \) where the interval is \([0, 6]\).

5. [15 points] Find the derivative of \( G(x) = \int_{x^4}^{x} e^{t^2} \, dt \) using part 1 of the Fundamental Theorem of Calculus.

**Solution:** \( G(x) = \int_{0}^{x} e^{t^2} \, dt + \int_{x^4}^{x} e^{t^2} \, dt = - \int_{0}^{x^4} e^{t^2} \, dt + \int_{0}^{x} e^{t^2} \, dt = -F(x^4) + F(x) \) where \( F(x) = \int_{0}^{x} e^{t^2} \, dt \) then the Fundamental Theorem of Calculus, Part 1 tells us that \( F'(x) = e^{x^2}. \) Now, using the Chain Rule, we have: \( G'(x) = -F'(x^4) 4x^3 + F'(x) = -4x^3 e^{x^8} + e^{x^2}. \)

6. [15 points each] Do both of the following.

1. Evaluate \( \int (2t + 1 + 2 \cos (2t + 1)) \, dt = t^2 + 1 + \sin (2t + 1) + C. \)

   **Solution:** \( \int (2t + 1 + 2 \cos (2t + 1)) \, dt = \int (2t + 1) \, dt + \int 2 \cos (2t + 1) \, dt = t^2 + t + \int 2 \cos (2t + 1) \, dt. \)

   Using the substitution \( u = 2t + 1, \ du = 2 \, dt \) on this last integral we get \( \int 2 \cos (2t + 1) \, dt = \int \cos (u) \, du = \sin (u) + C = \sin (2t + 1) + C. \)

2. Evaluate \( \int \frac{(\ln(x+1))^2}{x+1} \, dx = \frac{(\ln(x+1))^2}{3} + C \)

   **Solution:** Using the substitution \( u = \ln (x + 1) \) we get \( du = \frac{1}{x+1} \, dx \) so that \( \int \frac{(\ln(x+1))^2}{x+1} \, dx = \int u^2 \, du = \frac{1}{3} u^3 + C = \frac{(\ln(x+1))^2}{3} + C. \)