Do any six (6) of the following problems

1. (15 points) Use the error bound formula for the Trapezoid Rule, \( |E_T| \leq \frac{M(b-a)^3}{12n^2} \) to estimate the minimum number of subintervals needed to approximate the integral \( \int_0^2 \sqrt{x+1} \, dx \) with an error of magnitude less than \( 10^{-4} \).

2. (15 points) Does the following integral converge or diverge? Show all work.
   \[
   \int_{-\infty}^{\infty} \frac{x \, dx}{(x^2+9)^{3/2}}
   \]

3. Below are six infinite series,

   (a) (2 points each) For five (5) of the six, state a reasonable test for checking for convergence or divergence and include a short sentence as to why that test is reasonable.

   (b) (10 points each) Choose three (3) of the series and determine if they converge or diverge. Show all details.

   i. \( \sum_{n=1}^\infty \frac{6^n+1}{7^n} \)
   
   ii. \( \sum_{n=1}^\infty \frac{7^n}{6^n+1} \)
   
   iii. \( \sum_{n=1}^\infty \frac{\ln(n)}{n} \)
   
   iv. \( \sum_{n=2}^\infty \frac{6n^3-10n^2+1000n}{n^3-1} \)
   
   v. \( \sum_{n=1}^\infty \left( \frac{n-1}{n} \right)^n \)
   
   vi. \( \sum_{n=1}^\infty \frac{n!}{(2n)!} \) [Be careful with the factorials.]

4. (15 points) Determine if the following series converges or diverges. If it converges, determine if the convergence is absolute or conditional.

   \( \sum_{n=1}^\infty (-1)^{n+1} \frac{\sin(n)}{n^4} \)

5. (10 points) Prove the following theorem.

   If \( \sum_{n=1}^\infty a_n \) is a convergent series of nonnegative numbers, then \( \sum_{n=1}^\infty \frac{a_n}{n} \) also converges.