Technology used: __________________________________________________ Directions:

- Be sure to include in-line citations every time you use technology.
- Include a careful sketch of any graph obtained by technology in solving a problem.
- Only write on one side of each page.
- When given a choice, specify which problem(s) you wish graded.

You must do this problem

Required Problem (10 points) Express the integrand of the following integral as a sum of partial fractions with undetermined coefficients. Do not solve for the coefficients or evaluate the integrals.

\[ \int \frac{x^{12} - 6x^5 + 7}{x^3(x + 3)^2(x^2 + 9)(x^2 + x + 5)^3} \, dx \]

Do any six (6) of the following problems

1. (15 points) Write an integral for the area of the surface generated by revolving the curve \( y = \cos(x), \) \(-\pi/2 \leq x \leq \pi/2\) about the \( x \)-axis. Do not evaluate the integral.

2. (15 points) The half-life of californium-252 is 2.645 years. How long will it take 95% of a sample’s radioactive nuclei to disintegrate?

3. (15 points) Find the center of mass of a thin plate covering the region between the \( x \)-axis and the curve \( y = \frac{2}{x^2}, 1 \leq x \leq 2,\) if the plate’s density at the point \( (x, y) \) is \( \delta(x) = x^2.\) If you prefer to have units, \( x \) is measured in centimeters, and mass is measured in grams.

4. (15 points) Use integration by parts to evaluate the integral

\[ \int \arctan(x) \, dx \]

5. (15 points) Evaluate the integral

\[ \int \sin^2(2\theta) \cos^3(2\theta) \, d\theta \]

6. (15 points) Find the length of the curve \( y = \ln(\sec(x)), 0 \leq x \leq \pi/4.\)

7. (15 points) Use a trigonometric substitution to evaluate the integral

\[ \int \frac{x^2}{x^2 + 4} \, dx \]

8. (15 points) Express the integrand of the following integral as a sum of partial fractions and evaluate the integrals

\[ \int \frac{x^2 + 2x + 1}{(x^2 + 1)^2} \, dx \]