Be sure to re-read the WRITING GUIDELINES rubric, since it defines how your project will be graded. In particular, you may discuss this project with others but **you may not collaborate on the written exposition of the solution.**

“It is by logic that we prove but by intuition that we discover.” (Henri Poincaré)

**Two Theorems to Prove**

Part 1. There is exactly one real number that can be written in the position marked by $\Box$ in Theorem (1) below that makes the statement true. Determine that number and prove the resulting theorem.

**Theorem 1 (1)** Given any two solutions $(\beta_1, \beta_2, \ldots, \beta_n)$ and $(\gamma_1, \gamma_2, \ldots, \gamma_n)$ of the linear equation $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$. If $(\beta_1 + \gamma_1, \beta_2 + \gamma_2, \ldots, \beta_n + \gamma_n)$ is also a solution. Then $b = \Box$.

Part 2. Prove the following theorem.

**Theorem 2 (2)** Given two solutions $(\beta_1, \beta_2, \ldots, \beta_n)$ and $(\gamma_1, \gamma_2, \ldots, \gamma_n)$ of the linear equation $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$. Then $(\beta_1 - \gamma_1, \beta_2 - \gamma_2, \ldots, \beta_n - \gamma_n)$ is a solution of the related linear equation $a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0$. 