1. [15 points] Do one (1) of the following.
   (a) Using Riemann sums, carefully explain why the formula for the method of slicing, \( \int_a^b A(x) \, dx \) gives the volume of a solid. Include the meaning of \( a, b, \) and \( A(x) \).
   (b) If \( \{a_n\} \) is an infinite sequence of numbers, fully describe the definition of what it means to say the infinite series \( \sum_{n=1}^{\infty} a_n \) converges to the number \( L \).

2. [15 points] Find the exact length of the curve given by \( x = \frac{1}{6} y^3 + \frac{1}{2} y^{-1} \) from \( y = 2 \) to \( y = 3 \).

3. [15 points] Solve the following initial value problem. Express your answer \( y \) as a function of \( x \).
   \[
   \sec(x) \frac{dy}{dx} = e^{y + \sin(x)}, \quad y(0) = 0
   \]

4. [15 points] Determine the exact sum of the convergent geometric series
   \[
   \sum_{n=2}^{\infty} \frac{(-1)^n}{5^n}
   \]

5. [15 points] Find the radius and interval of convergence of the following power series. Also determine any values of \( x \) for which the series converges conditionally.
   \[
   \sum_{n=0}^{\infty} \frac{(-1)^n(x - 1)^{n+2}}{2n + 1}
   \]

6. [15 points] Determine the Taylor Series for the function \( f(x) = (x + 3)^{-2} \) when \( a = -1 \).

7. [15 points each] By hand (without using a calculator or table of integrals), evaluate two (2) of the following integrals
   (a) \( \int \frac{\ln(t+1)^2}{t+1} \, dt \)
   (b) \( \int x^2 e^{4x} \, dx \)
   (c) \( \int \sqrt{1 - 9t^2} \, dt \)
   (d) \( \int \frac{4x^2}{(x-1)(x^2 + 2x + 1)} \, dx \)

8. [15 points each] Do two (2) of the following:
   (a) Find the radius of convergence of the series
   \[
   \sum_{n=1}^{\infty} \frac{2 \cdot 5 \cdot 8 \cdots (3n - 1) x^n}{2 \cdot 4 \cdot 6 \cdots (2n)}
   \]
   (b) Prove that if all of the terms \( a_n \) are positive and the series \( \sum_{n=1}^{\infty} a_n \) converges, then the series \( \sum_{n=1}^{\infty} a_n^2 \) also must converge.
   (c) Prove the theorem that absolute convergence implies convergence. More specifically, prove that if the series \( \sum_{n=1}^{\infty} |a_n| \) converges then so does the series \( \sum_{n=1}^{\infty} a_n \).
Useful Information

Taylor’s Formula
If $f$ has derivatives of all orders in an open interval $I$ containing the number $a$, then for each positive integer $n$ and for each $x$ in $I$, we have $f(x) = P_n(x) + R_n(x)$ where $P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)(x-a)^k}{k!}$ and $R_n(x) = \frac{f^{(n+1)}(c)(x-a)^{n+1}}{(n+1)!}$ for some number $c$ between $a$ and $x$.

Frequently Used Taylor Series

- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, for $|x| < 1$
- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, for $|x| < \infty$
- $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$, for $|x| < \infty$
- $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$, for $|x| < \infty$
- $\ln(1 + x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$, for $-1 < x \leq 1$