Technology used: 

Directions: Be sure to show all of your work. Include a careful sketch of any graph obtained by technology in solving a problem.  **Only write on one side of each page.**

The Problems

1. (15 points) Do **one** of the following.

   (a) A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 10 cm by 22 cm by cutting out equal squares of side $x$ at each corner and then folding up the sides. Express the volume $V$ of the box as a function of $x$. Include the domain of your function.

   (b) A closed box with a square base is to have a volume of 300 cubic feet. The material for the top and bottom of the box costs $2.00 per square foot and the material for the sides costs $1.00 per square foot. Express the cost of building the box as a function of the length of its base. Include the domain of your function.

2. (15 points each) Do **two** of the following.

   (a) Suppose the graph of a function $f$ is given. Write equations for the graphs that are obtained from the graph of $f$ as follows.

      i. Shift 3 units upward and 4 units to the left.

      ii. Reflect about the $y$-axis then stretch vertically by a factor of 2.

   (b) Graph $f(x) = \arcsin(2x)$, not by plotting points or using your calculator, but by starting with the graph of $g(x) = \sin(x)$ and then applying the appropriate transformations.

   (c) Write the equation for the graph that is obtained by reflecting the graph of $y = x^3$ about the line $x = 2$.

3. (15 points) Find the exact value of

   (a) $\sin\left(\arccos\left(\frac{\sqrt{5}}{4}\right)\right)$

   (b) $\sec(\arctan(x))$

4. (15 points) Given $f(x) = \frac{x + 7}{x - 5}$, $x \neq 5$. Find $f^{-1}$, if it exists.

5. (15 points) Evaluate the following limits or show they do not exist.

   (a) 
   $$\lim_{x \to 2} \frac{x^2 + 3x - 10}{3x^2 + 5x - 7}$$
(b) \[
\lim_{x \to 2} \frac{|x - 2|}{x - 2}
\]

6. (10 points) Provide an \( \varepsilon - \delta \) proof that \( \lim_{x \to 3} \frac{2x^2 - 7x + 3}{x - 3} = 5 \).